Appendix I

Rough Estimation of Energy and Water Relations for Variously Treated Rooftops in New York City—Hypothetical Calculations

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Energy Relations

We begin with the energy-balance equation (Hillel, 1998):

$$S = (J_s + J_a)(1 - a) + (J_{li} - J_{lo}) - A - LE \qquad (1)$$

Where S is downward heat flux into the ground surface (i.e., into the building under a flat roof). Js is the incoming flux of shortwave radiation directly from the sun, Ja is the shortwave diffuse radiation from the atmosphere (sky), Jli is the incoming long-wave radiation flux from the sky, Jlo is the outgoing longwave radiation, a is the albedo, A is the sensible heat flux transmitted from the surface to the air, and LE is the evaporative heat flux, a product of the evaporative rate E and the latent heat per unit quantity of water evaporated, L.

Assume, for simplicity, that the diffuse short-wave and longwave sky radiations (Ja and Jli, respectively) are negligible. (They are often small in comparison to the other fluxes.). Hence

$$S = J_s(1 - a) - J_{lo} - A - LE \qquad (2)$$

For a bare roof (no vegetation), we assume that there is no evaporative heat flux, since, for the most part, there is no standing water; thus, LE = 0.

We estimate S from the following considerations:

The flux of radiant energy received at the outer envelope of the atmosphere (known as the "solar constant" is about 1400 Watts per square meter perpendicular to the incoming radiation. The flux of solar radiation actually reaching any portion of the earth's surface varies according to latitude, season, cloudiness, and atmospheric turbidity.

Assume that the warm season (the period of air-conditioning in New York) lasts 140 days, of which 100 days are bright sunny days. If the flux of solar radiation on bright days averages about 50% of the solar constant during 10 hours (of the total day-length of 14 hours), then:

Js = 0.5(1400 Watts per hour)(10 hours) = 7000 Watt-hours per day = 7 Kilowatt-hours per square meter per day (7 KWH/day)(100 bright sunny days per year) = 700 KWH per square meter per year

Black Roof Estimation

We now do the calculation for a hypothetical black (smooth and flat) roof, for which the albedo a = 0.1, evapotranspiration LE = 0, Jl = 0.1 of Js, and sensible heating of the air A = 0.1:

S = [(1 − 0.1) − (0.1 + 0.1)]Js = (1 − 0.3)Js = (0.7)(700 KWH/yr) = 500 KWH per square meter per year

The total area of flat roofs in New York's five boroughs is 21,250 acres, equal to 85,000,000 square meters. Hence the total annual heat load into black-roofed buildings is:

S = (500)(85,000,000) = 42,500,000,000 KWH

Assume, for the sake of argument, that the above heat load must be dissipated by air-conditioning, that the air-conditioners are 50% energy-efficient, and that the cost of electricity is $0.1 per KWH. Then the annual cost of air-conditioning for all black-roofed buildings in New York would be:

($0.1/0.5)(42.5 BKH) = $8.5 billion dollars

White Roof Estimation

For comparison, let us assume that all the roofs of New York were painted white, so that the albedo (a) would be increased from 0.1 to 0.7. Because white surfaces would be cooler than black surfaces, Jlo (emitted long-wave radiation) and A (sensible heating of the air) would then both reduced to, say, 0.05. So, for white roofs,

S = (1 – 0.7)Js – (0.05 + 0.05)Js = (1 – 0.8)Js
S = (0.2)(700) = 140 KWH per square meter per year

This is less than one-third of the heat load affecting the buildings under the black roofs.

Again we multiply the last figure by the total area of roofs in New York City to obtain the total heat load under white roofs:

S = (140)(85,000,000) = 11.7 billion KWH per year

Assuming the same 50% efficiency for air-conditioners, we estimate the cost of cooling white-topped buildings to be

($0.1/0.5)(11.7 BKH) = $2.34 billion dollars

Green Roof Estimation

Now assume that a shallow layer of "soil" (or of a porous, particulate material simulating soil) covers all the roofs, constituting a medium for the growth of a dense stand of active vegetation. To grow in a very shallow rooting zone and to survive repeated dry spells, the vegetation to be grown will necessarily be of the drought-tolerant "xerophytic" type.

The albedo value of vegetation may vary between 0.15 and 0.4. Xerophytes generally exhibit higher values of albedo. We therefore assume a mean value of 0.3.

Fully active vegetation, growing in a medium well-endowed with water, typically transpire at nearly all (say, 80%) of the meteorologically imposed potential evapo-transpiration rate. In our case, because of the expected occurrence of repeated dry spells between successive rainfall events, we may assume that the vegetation transpires, on average, at a rate roughly equal to 40% of potential evapotranspiration.

Figures can be obtained for the potential evapotranspiration prevailing in New York, and its variation over the seasons. In our case, we may use the starting assumption that the potential evapotranspiration is roughly equal to the flux of incoming short-wave solar radiation. Accordingly, the value of the term LE in the energy balance equation = 0.4Js.

Because of its cooling effect on the surface, the process of transpiration reduces the Jlo and the A terms to perhaps 0.05Js. Therefore,

S = Js(1 - 0.3) - (0.05 + 0.05 + 0.4)Js
= 700(1 - 0.8) = 140 KWH per square meter

These considerations suggest that the effect of vegetation on the energy balance might be similar to the effect of whitening the inert surface. However, the presence of vegetation may offer the additional advantages of aesthetics, as well as reducing the quantity and intensity of storm-water runoff.

Water Relations of Green Roofs

To estimate the effect of "green roofs" (i.e., roofs covered with a shallow layer of a porous medium in which low plants are grown), assume an annual precipitation of about 1000 mm (40 inches). Now assume that the annual rate of evapotranspiration from a dense stand of xerophytes is likely to be about 50% of the potential evaporation, which we estimate to be about equal to the annual precipitation.

For a total roof area of 85 million square meters, the volume of runoff from bare roofs is about 85 million cubic meters. That amount can probably be reduced by half if the roofs are covered with vegetation.

The amount of water absorbed and stored in the growth medium (an artificial soil serving as the rooting medium for the plants to be grown) during each spell of rain (rain event, or rainstorm) depends on the porosity and depth (thickness) of the porous medium to be used. (By the term "spell of rains," we refer to a sequence of rainstorms that is not interrupted by a period of evapotranspiration).

If the depth of the medium is, say, 10 cm and the porosity 50%, then the so-called "water-holding capacity" is 5 cm. Each square meter can therefore hold 0.05 cubic meter of water (50 liters). Rain spells that do not exceed 5 cm (2 inches), if they follow a period of dryness (in which the "soil" moisture had been depleted), will produce no runoff at all. The greater the depth of the growth medium and the greater its porosity, the greater will be its effect in preventing runoff from the smaller storms and reducing runoff from the larger ones.

Hence we need to consider the local pattern of rainfall in order to assess the probable number of rain spells or storms that will produce no runoff from a given medium, and the possible reduction in the volume of runoff resulting from the rainstorms (or rain spells) that will exceed the available porous-medium storage and that will produce runoff of varying amounts.

Let us assume, hypothetically, that in a typical year there occur 50 spells of rain (one per week on average), each lasting two days, with a mean amount of 20 mm per rain spell. That amount can readily be absorbed into the envisaged porous substrate. Assume, furthermore, that the average potential evapotranspiration during the dry spell between rains is 4 mm per day. If the average dry spell between rains lasts about 5 days, and if the plants transpire at the full potential rate, then the moisture reserve in the growth medium will be depleted within five days. If, however, the rate of transpiration is below the potential evaporation rate (as is typical of the transpiration rate of xerophytes), then the moisture reserve may only be depleted in part. Clearly we also need to know much more about the specific water relations of the plants being considered for the green roof project to obtain a more accurate estimate.

In any case, it seems reasonable that a stand of plants growing in a porous medium capable of absorbing rainwater during spells of rain and of retaining that water for the subsequent use of plants, may well reduce the number of runoff-producing events (perhaps by half). Such a stand may also reduce the volume of runoff produced during spells of particularly heavy rainfall. Altogether, it seems reasonable that the presence of an absorptive layer of soil-like material, in which an actively transpiring stand of vegetation is growing, may well reduce storm-water runoff by some 50%.

For an area of 85,000,000 square meters, under a rainfall regime of 1,000 mm per season, the envisaged reduction amounts to some 42 million cubic meters.

General Comments and Caveats

The figures given above are very crude preliminary estimations, which need to be improved as more exact quantitative knowledge is obtained regarding the relevant variables and parameters. They can, however, serve (at least temporarily) as bench marks against which other estimates can be compared. Eventually, preliminary estimates will be supplanted by actual, measured data.

Reference

Hillel, D. 1998. *Environmental Soil Physics.* Academic Press. San Diego, CA. 771 pp.